

Matrix Model in a Class of Time Dependent Supersymmetric Backgrounds

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Abstract

We discuss the matrix model in a class of 11D time dependent supersymmetric backgrounds as obtained in [9]. We construct the matrix model action through the matrix regularization of the membrane action in the background. We show that the action is exact to all order of fermionic coordinates. Furthermore We discuss the fuzzy sphere solutions in this background.

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1 Introduction

It is very important to understand string theory in the time dependent background because this issue is related to some fundamental questions in quantum gravity. One question is the resolution of the cosmological singularities. Near the big-bang or big-crunch singularity, the quantum effects should play an important role and a quantum gravity description is needed. However, despite of many efforts in the past decades, we are still far from understanding the issue clearly. To address the issue, we have to decide what the right degrees of freedom are to describe the physics there. If the string coupling is small, we might hope that the perturbative string is suitable. One class of models, called null orbifold, have been constructed to investigate this possibility [1]. These models keep part of the supersymmetries and are solvable perturbatively. Unfortunately, it turned out that these time dependent orbifold models are unstable to large back reaction because of blue shifting of modes in these background[2]. Quite recently, E. Silverstein et al propose that a closed string tachyon condensate smooths out the singularity by consistently massing up the degrees of freedom of the system[3].

If the string coupling is large near the singularity, one must take the non-perturbative string effects seriously. Very recently, the authors in [4] raised the idea of matrix big bang. They considered a type IIA theory in a null linear dilaton background which preserves one-half of the original supersymmetry. In this time-dependent background, the string coupling becomes large near the big-bang singularity. In [4], the authors proposed a dual matrix string which is a two-dimensional Super-Yang-Mills theory on the Milne orbifold to describe the big bang where the Yang-Mills coupling becomes weak. In short, the matrix degrees of freedom, rather than the point particle or the perturbative string, describe the physics near big-bang singularity. Following [4], many authors discussed the generalization of their background [5]-[16].

In [9], a large class of rather general time-dependent configurations have been found in M-Theory. These configurations keep sixteen supersymmetries, with killing spinor satisfying $\Gamma^+\epsilon = 0$, and has a null Killing field. Moreover, it has been proved that such configurations generally have no supernumerary supersymmetries. As a consequence, the corresponding matrix model constructed by the DCLQ prescription has no linearly realized supersymmetry.

One subtle point in [9] is that the construction of the matrix model follows the route of the weak field approximation [17]. However, in the early time the configurations turn out to be far from flat. It seems that the matrix

model construction in [9] is in doubt. Generically the matrix model in curved backgrounds is a subtle issue. Besides the weak field approximation, there is another way to get the matrix model action. The way is to start from the supermembrane action embedded in a 11D curved background, and then transfer to the matrix model action through matrix regularization. This has been proved successful in recovering the BFSS matrix model from the membrane in the flat spacetime [18, 19] and BMN matrix model from the membrane in the 11D maximally supersymmetric plane-wave background [20, 21]. However it should be noticed that in a general curved background, the membrane action could only be obtained order by order of fermionic coordinates θ . Up to order of θ^2 , the explicit form of the action has been worked out in [22].

In this short note, we would like to construct the matrix model action of the configurations found in [9], following the route of matrix regularization of supermembrane action. In our case, we manage to get the exact membrane action to all order of θ . Moreover, we will discuss the evolution of the fuzzy sphere solutions of the model. We will find that the radius of the fuzzy sphere shrinks to zero in a big-bang like evolution while it grows without limit in a big crunch like evolution.

2 The Matrix Model

Let us first give a short review of the background. The metric of our background is as follows:

$$\begin{aligned}
ds^2 &= 2e^{r_0 u} du dv + \sum_i c_i e^{r_i u} (x^i)^2 (du)^2 + \sum_i e^{r_i u} (dx^i)^2 \\
&+ \sum_{ij} A_{ij}^0 e^{(r_i + r_j)u/2} x^j dx^i du,
\end{aligned} \tag{1}$$

where

$$A_{ij}^0 = -A_{ji}^0 = \text{const}, \tag{2}$$

and r_0, r_i are all constants, too. We also have a four-form field strength

$$F_{u123} = e^{(r_1 + r_2 + r_3)u/2} f^0, \quad f^0 = \text{const}. \tag{3}$$

Our convention is as follows: we use x^μ for the curved space coordinates with

$$x^\mu = (x^u, x^v, x^i) \equiv (u, v, x^i), \tag{4}$$

where $u \equiv x^u = (x^{\mu=10} + x^{\mu=0})/\sqrt{2}$, and $v \equiv x^v = (x^{\mu=10} - x^{\mu=0})/\sqrt{2}$, and $i = (1, \dots, 9)$. Similarly, We use x^r to represent tangent space coordinates with

$$x^r = (x^+, x^-, x^I), \quad (5)$$

where $x^+ = (x^{r=10} + x^{r=0})/\sqrt{2}$, and $x^- = (x^{r=10} - x^{r=0})/\sqrt{2}$, and $I = (1, \dots, 9)$.

The background keeps sixteen “standard” supersymmetries characterized by Killing spinor satisfying $\Gamma^+ \epsilon = 0$. There is no supernumerary supersymmetry in this case. This indicates that there is no linearly realized supersymmetries in the embedded supermembrane action and hence in the matrix model action. Another remarkable fact is that there exist a null Killing vector in the background. And also the Ricci tensor and the field strength have no lower index in v and no dependence on v .

We will begin to derive the matrix model in this background following [22]. The supermembrane action is:

$$S[Z(\xi)] = \int d^3\xi [-\sqrt{-g(Z(\xi))} - \frac{1}{6} \epsilon^{abc} \Pi_a^A \Pi_b^B \Pi_c^C B_{CBA}(Z(\xi))], \quad (6)$$

where $Z^A(\xi) = (x^\mu(\xi), \theta(\xi))$ is the curved superspace coordinates, $g_{ab} = \Pi_a^\mu \Pi_b^\nu g_{\mu\nu} = \Pi_a^r \Pi_b^s \eta_{rs}$ is the induced metric, $\eta_{rs} = \text{diag}(-1, 1, \dots, 1)$ is the 11-d Lorentz metric, and $\xi^a = (\xi^0, \xi^1, \xi^2) = (\tau, \xi^\alpha)$, $\alpha = 1, 2$ represent the coordinates on the world volume. Here Π_a^A are the supervielbein pullback, B_{ABC} are the super three-potential. In [22], the authors have obtained the expression of these two quantities in terms of component fields to order θ^2 of fermionic coordinates. In our case, the gravitino is zero, so the supervielbein pullback is:

$$\begin{aligned} \Pi_a^r &= \partial_a Z^A E_A^r \\ &= \partial_a x^\mu (e_\mu^r - \frac{1}{4} \bar{\theta} \Gamma^{rst} \theta \omega_{\mu st} + \bar{\theta} \Gamma^r \Omega_\mu \theta) + \bar{\theta} \Gamma^r \partial_a \theta + \mathcal{O}(\theta^3), \end{aligned} \quad (7)$$

where $\omega_{\mu st}$ is the spin connection, and

$$\Omega_\mu = \frac{1}{288} F_{\nu\rho\sigma\lambda} (\Gamma_\mu^{\nu\rho\sigma\lambda} + 8 \Gamma^{\nu\rho\sigma} \delta_\mu^\lambda). \quad (8)$$

The super three-potential pullback is:

$$\begin{aligned}
& - \frac{1}{6} \Pi_a^A \Pi_b^B \Pi_c^C B_{CBA} \\
& = \frac{1}{6} \epsilon^{abc} \partial_a x^\mu \partial_b x^\nu \partial_c x^\rho \left[C_{\mu\nu\rho} + \frac{3}{4} \bar{\theta} \Gamma_{rs} \Gamma_{\mu\nu} \theta \omega_\rho^{rs} - 3 \bar{\theta} \Gamma_{\mu\nu} \Omega_\rho \theta \right] \\
& - \epsilon^{abc} \bar{\theta} \Gamma_{\mu\nu} \partial_c \theta \left[\frac{1}{2} \partial_a x^\mu (\partial_b x^\nu + \bar{\theta} \Gamma^\nu \partial_b \theta) + \frac{1}{6} \bar{\theta} \Gamma^\mu \partial_a \theta \bar{\theta} \Gamma^\nu \partial_b \theta \right] + \mathcal{O}(\theta^3), \quad (9)
\end{aligned}$$

where $C_{\mu\nu\rho}$ is the three-form potential. To simplify the action, we go to light-cone gauge:

$$x^u = u = \tau. \quad (10)$$

And because of the κ -symmetry of the action, we can also impose an additional gauge [22]

$$\Gamma^+ \theta = 0. \quad (11)$$

We further decompose our 11D gamma matrices Γ^I using 9D matrices γ^I as follows:

$$\Gamma^I = \gamma^I \otimes \sigma_3, \quad (I = 1, \dots, 9), \quad (12)$$

$$\Gamma^0 = 1 \otimes i\sigma_1, \quad (13)$$

$$\Gamma^{11} = -1 \otimes \sigma_2, \quad (14)$$

$$\Gamma_- = \Gamma^+ = \frac{1}{\sqrt{2}} (\Gamma^0 + \Gamma^{11}), \quad (15)$$

$$\Gamma_+ = \Gamma^- = \frac{1}{\sqrt{2}} (-\Gamma^0 + \Gamma^{11}). \quad (16)$$

Then θ can be decomposed as:

$$\theta = \frac{1}{2^{1/4}} (\psi^T, 0)^T, \quad (17)$$

$$\bar{\theta} = \frac{1}{2^{1/4}} (0, -\psi^T). \quad (18)$$

Using the formula of [22], we can only derive the following formulae up to $\mathcal{O}(\theta^2)$. But as we will argue at the end of this section, our matrix model is exact to all orders of θ . So we omit the terms higher than θ^2 in the following formulae.

Plugging the above expressions into the action, for the metric we get

$$g_{\alpha\beta} = \sum_i e^{r_i\tau} \partial_\alpha x^i \partial_\beta x^i, \quad (19)$$

$$\begin{aligned} g_{00} &= \sum_i c_i e^{r_i\tau} (x^i)^2 - \sum_{IJ} \frac{i}{4} e^{r_0\tau/2} A_{IJ}^0 \psi^T \gamma^{IJ} \psi - \frac{i}{3} e^{r_0\tau/2} f^0 \psi^T \gamma^{123} \psi \\ &+ 2e^{r_0\tau} \partial_0 v + 2ie^{r_0\tau/2} \psi^T \partial_0 \psi + \sum_{ij} A_{ij}^0 e^{(r_i+r_j)\tau/2} x^j \partial_0 x^i \\ &+ \sum_i e^{r_i\tau} (\partial_0 x^i)^2. \end{aligned} \quad (20)$$

We don't need the explicit form of $u_\alpha \equiv g_{0\alpha}$ in the later calculations, and we only need to know that it depends on \dot{X}^i . For the super three-form potential term, we get

$$\begin{aligned} &- \frac{1}{6} \epsilon^{abc} \Pi_a^A \Pi_b^B \Pi_c^C B_{CBA}(Z(\xi)) \\ &= -i \sum_{I,i} \psi^T \gamma^I \{x^i, \psi\} e^{(r_0+r_i)\tau/2} \delta_i^I \\ &- \frac{1}{2} \sum_{i,j=1,2,k=3} \{x^i, x^j\} x^k \epsilon_{ijk} f^0 e^{(r_i+r_j+r_k)\tau/2}, \end{aligned} \quad (21)$$

where

$$\{A, B\} = \epsilon^{\alpha\beta} \partial_\alpha A \partial_\beta B. \quad (22)$$

Now, we decompose $g = \det(g_{ab})$ as following:

$$g = -\Delta \bar{g}, \quad (23)$$

where

$$\bar{g}_{\alpha\beta} = g_{\alpha\beta}, \quad (24)$$

$$\bar{g} = \det(\bar{g}_{\alpha\beta}), \quad (25)$$

$$\bar{g}^{\alpha\beta} \bar{g}_{\beta\gamma} = \delta_\gamma^\alpha, \quad (26)$$

$$\Delta = -g_{00} + u_\alpha \bar{g}^{\alpha\beta} u_\beta. \quad (27)$$

To solve the constraints, we go to the Hamiltonian formalism. We get the

expression for the canonical momentum of the X^i, v , and ψ :

$$P_v = P^u = e^{r_0\tau} \sqrt{\frac{\bar{g}}{\Delta}}, \quad (28)$$

$$P_\psi = ie^{r_0\tau/2} \sqrt{\frac{\bar{g}}{\Delta}} \psi^T = ie^{-r_0\tau/2} \psi^T P^u, \quad (29)$$

$$P_i = \sqrt{\frac{\bar{g}}{\Delta}} (e^{r_i\tau} \partial_0 x^i + \frac{1}{2} \sum_j A_{ij}^0 e^{(r_i+r_j)\tau/2} x^j - e^{r_i\tau} \partial_\alpha x^i \bar{g}^{\alpha\beta} u_\beta). \quad (30)$$

After the Legendre transformation:

$$\mathcal{H} = \sum_i P_i \dot{x}^i + P_v \dot{v} + P_\psi \dot{\psi} - \mathcal{L}, \quad (31)$$

we have the hamiltonian density:

$$\begin{aligned} \mathcal{H} = & \sum_i \frac{P_i^2}{2p^\tau} e^{(r_0-r_i)\tau} + \frac{e^{r_0\tau}}{4P^u} \sum_{ij} e^{(r_i+r_j)\tau} \{x^i, x^j\}^2 \\ & - \frac{1}{2} \sum_{ij} A_{ij}^0 e^{(r_j-r_i)\tau/2} x^j P_i + i \sum_{I,i} \psi^T \gamma^I \{x^i, \psi\} e^{(r_0+r_i)\tau/2} \delta_i^I \\ & + \frac{1}{2} \sum_{i,j=1,2,k=3} \{x^i, x^j\} x^k \epsilon_{ijk} f^0 e^{(r_i+r_j+r_k)\tau/2} \\ & + \frac{1}{2} e^{-r_0\tau} P^u \left[- \sum_i c_i e^{r_i\tau} (x^i)^2 + \frac{i}{4} \sum_{IJ} e^{r_0\tau/2} A_{IJ}^0 \psi^T \gamma^{IJ} \psi \right. \\ & \left. + \frac{i}{3} e^{r_0\tau/2} f^0 \psi^T \gamma^{123} \psi + \frac{1}{4} \sum_{ijk} A_{ij}^0 A_{ik}^0 e^{(r_j+r_k)\tau/2} x^j x^k \right]. \quad (32) \end{aligned}$$

This Hamiltonian density can be derived from a Lagrangian density consist-

ing of only physical degrees of freedom x^i and ψ of the following form:

$$\begin{aligned}
\mathcal{L} = & \sum_i \frac{P^u}{2} e^{(r_i - r_0)\tau} (D_\tau x^i)^2 + \frac{P^u}{2} \sum_{ij} A_{ij}^0 e^{(\frac{r_i + r_j}{2} - r_0)\tau} x^j D_\tau x^i \\
& + \frac{P^u}{2} \sum_i c_i e^{(r_i - r_0)\tau} (x^i)^2 - \frac{e^{r_0\tau}}{4P^u} \sum_{ij} e^{(r_i + r_j)\tau} \{x^i, x^j\}^2 \\
& - \frac{1}{2} \sum_{i,j=1,2,k=3} \{x^i, x^j\} x^k \epsilon_{ijk} f^0 e^{(r_i + r_j + r_k)\tau/2} \\
& + i P^u e^{-r_0\tau/2} \psi^T D_\tau \psi - \frac{i}{6} P^u e^{-r_0\tau/2} f^0 \psi^T \gamma^{123} \psi \\
& - \frac{i}{8} P^u \sum_{IJ} e^{-r_0\tau/2} A_{IJ}^0 \psi^T \gamma^{IJ} \psi - i \sum_{I,i} \psi^T \gamma^I \{x^i, \psi\} e^{(r_0 + r_i)\tau/2} \delta_i^I, \quad (33)
\end{aligned}$$

where D_τ is the covariant derivative with respect to an auxiliary gauge field A_0 .

Now, let us do the usual matrix regularization:

$$x^i \rightarrow X_{N \times N}^i, \quad (34)$$

$$\psi \rightarrow \psi_{N \times N}, \quad (35)$$

$$P^u \int d^2\sigma \rightarrow \frac{1}{R} Tr, \quad (36)$$

$$\{, \} \rightarrow -i[,], \quad (37)$$

in the above membrane action, and we finally obtain the matrix model action:

$$\begin{aligned}
S = & \int d\tau Tr \left(\sum_i \frac{1}{2R} e^{(r_i - r_0)\tau} (D_\tau X^i)^2 + \frac{1}{2R} \sum_{ij} A_{ij}^0 e^{(\frac{r_i + r_j}{2} - r_0)\tau} X^j D_\tau X^i \right. \\
& + \frac{1}{2R} \sum_i c_i e^{(r_i - r_0)\tau} (X^i)^2 + \frac{R}{4} e^{r_0\tau} \sum_{ij} e^{(r_i + r_j)\tau} [X^i, X^j]^2 \\
& + \frac{i}{2} \sum_{i,j=1,2,k=3} [X^i, X^j] X^k \epsilon_{ijk} f^0 e^{(r_i + r_j + r_k)\tau/2} \\
& + \frac{i}{R} e^{-r_0\tau/2} \psi^T D_\tau \psi - \frac{i}{6R} e^{-r_0\tau/2} f^0 \psi^T \gamma^{123} \psi \\
& \left. - \frac{i}{8R} \sum_{IJ} e^{-r_0\tau/2} A_{IJ}^0 \psi^T \gamma^{IJ} \psi - \sum_{I,i} \psi^T \gamma^I [X^i, \psi] e^{(r_0 + r_i)\tau/2} \delta_i^I \right). \quad (38)
\end{aligned}$$

Although this action seems a bit cluttered, it can be cast into a canonical form by rescalings [23]. The bosonic part of the action is the same as the one raised in [9]¹, while the fermionic part is different by the prefactors. The main discrepancy in the fermionic actions comes from the g^{uv} factor. We suspect that such factors have not been incorporated properly in the weak field approximation. We believe that the treatment in this paper is more convincing.

Although we have derived this matrix model using formulae of [22] that are only exact to order θ^2 , we will now argue that it is in fact exact to all orders of θ . The argument is quite similar to that in [24], in which the authors argued that his matrix model on a pp-wave background is exact to all orders of θ . Similar argument has been used in the discussion of the Green-Schwarz string action in a class of plane-wave background [25]. The main points are as follows. First notice that the supervielbein pullback $\Pi_a^r = \partial_a Z^A E_A^r$ is linear in $\partial_a X^\mu$, while E_A^r is constituted with other quantities. It can be seen from their explicit form that these other quantities, θ , Γ^r , Ricci tensor, Ω_μ , and field strength et.al. have no lower curved spacetime index v , and hence no upper curved spacetime index u . Also from the form of the metric, we notice that the only spin connections $\omega_{\mu\nu\rho} = \omega_\mu^{rs} e_{r\mu} e_{s\nu}$ with lower curved spacetime index v is ω_{uv} , and the only geometrical object with lower index v constructed from the vielbein $e_\mu^r e_{r\nu}$ and their derivatives must have the lower index uv appearing at the same time. Hence, although these two quantities can have upper curved spacetime index u , they must also have lower curved spacetime index u at the same time. On the other hand, the nonvanishing bilinear fermionic terms $\bar{\theta}\Gamma^{rst\dots}\theta$ always have one and only one Γ^- and no Γ^+ due to the gauge condition $\Gamma^+\theta = 0$. The upper tangent space index $r = -$ require an upper curved spacetime index $\mu = u$ coming from other geometrical quantities because the only nonzero vielbein with a lower tangent index $r = -$ is e_{-u} . Such an index cannot be cancelled by the above mentioned quantities except $\partial_a X^u$. For example, the other two possible quantities with the upper curved index u must carry the lower curved spacetime index u at the same time. So the net result is to leave a lower curved index u . This index can only be cancelled by $\partial_a X^u$. But due to the linearity in $\partial_a X^\mu$, one at most has bilinear θ terms in Π_a^A . Also

¹While completing the manuscript, we realized that in [9], there exists two typos in the bosonic action. One is a sign difference, the other is due to the overcounting of the background 3-form potential. After fixing them, the bosonic action in [9] agrees with the above one.

as a consequence, the super three-potential pullback term can only have bilinear θ terms. This is due to the antisymmetric nature of ϵ^{abc} and the fact that bilinear θ term in Π_a^A must be proportional to $\partial_a X^u$. In short, the expressions (7,9) have vanishing higher order terms and so are exact to all order of θ . Therefore the matrix model (38) is exact to all orders of fermionic coordinates.

3 The Fuzzy Sphere Solution

We would like to discuss the fuzzy sphere solution of the classical equation of motion derived from the matrix model action. To investigate the simplest situation, consider the matrix model in the sector:

$$X^4 = X^5 = \dots = X^9 = 0, \psi = 0. \quad (39)$$

To further simplify the problem, we restrict ourselves to symmetric case with:

$$r_1 = r_2 = r_3 = r, c_1 = c_2 = c_3 = c. \quad (40)$$

We want to find solution of the form:

$$X^a(\tau) = S(\tau)J^a, a = 1, 2, 3, \quad (41)$$

where J^a is N dimensional representation of $SU(2)$. Use

$$Tr \sum_a (J^a)^2 = \frac{N(N-1)}{4}, \quad (42)$$

and

$$[J^a, J^b] = i\epsilon^{abc} J^c. \quad (43)$$

We finally get

$$\frac{d^2 S}{d\tau^2} + (r - r_0) \frac{dS}{d\tau} + 2R^2 e^{(2r_0+r)\tau} S^3 + Rf^0 e^{(r_0+r/2)\tau} S^2 - cS = 0. \quad (44)$$

We change this equation to be dimensionless by introducing two dimensionless variables:

$$t = r_0\tau, S(\tau) = R\tilde{S}(t), \quad (45)$$

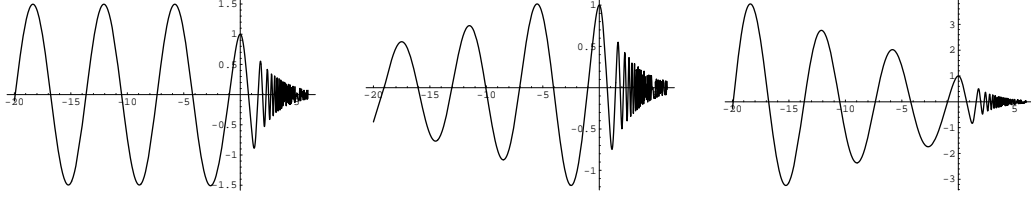


Figure 1: Fuzzy sphere solution with $\hat{r} = 1, 0.8, 1.1$ respectively.

We insert appropriate powers of l_p and further introduce the other dimensionless variables as follows:

$$\hat{r} = \frac{r}{r_0}, \hat{c} = \frac{c}{r_0^2}, \hat{f}^0 = \frac{f^0}{2\sqrt{2}r_0}, A = \frac{\sqrt{2}R^2}{r_0 l_p^3}. \quad (46)$$

The equation then becomes:

$$\frac{d^2 \tilde{S}}{dt^2} + (\hat{r} - 1) \frac{d\tilde{S}}{dt} + A^2 e^{(2+\hat{r})t} \tilde{S}^3 + 2\hat{f}^0 A e^{(1+\hat{r}/2)t} \tilde{S}^2 - \hat{c} \tilde{S} = 0. \quad (47)$$

To investigate the behavior of fuzzy sphere solution as the time evolves, we have numerically solved this equation. We chose initial conditions as:

$$\tilde{S}(0) = 1, \tilde{S}'(0) = 0, \quad (48)$$

and choose the dimensionless parameters as:

$$\hat{c} = -1, \hat{f}^0 = 1, A = 1. \quad (49)$$

The behavior for $\hat{r} = 1, 0.8, 1.1$ respectively is shown in figure 1. We see that the behavior is similar to that of [8], i.e. the radius of the fuzzy sphere shrinks to zero at late times as the spatial dimensions expand larger and larger in a big-bang like evolution. This is expected, as when the spatial dimensions expand larger, the effect of non-Abelian degrees of freedom of the matrix model become less important. The above is for the case $\hat{r} > 0$, for the case $\hat{r} < 0$, the evolution is different. In this case, the evolution is big-crunch like and we expect that as the spatial dimensions collapse, the effect of the non-Abelian degrees of freedom of the matrix model will become more and more important. We see this behavior in figure 2 that the radius of the fuzzy sphere grows as time evolves.

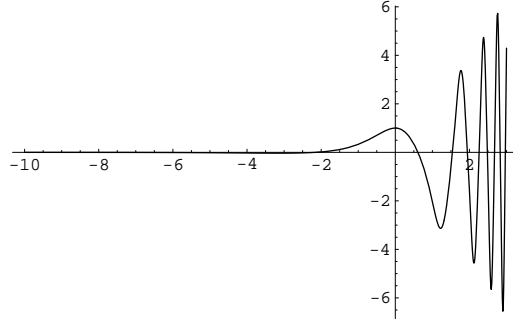


Figure 2: Fuzzy sphere solution with $\hat{r} = -1$.

The above behaviors at late time are also seen in the cases with other parameters choosing different values as far as they do not change their signs. Also the behaviors are the same if we change the initial condition. We also investigated the more general cases with non-symmetric metric, i.e. with different c_i 's and different r_i 's. Again, the late time behaviors are the same. So we conclude that our fuzzy sphere solutions are reasonable, and that the matrix model we derived is also reasonable.

4 Conclusions and Discussions

In this note, we studied the matrix model action in a class of the supersymmetric time-dependent backgrounds. We first discussed the membrane action in the background and then through matrix regularization we obtained the corresponding matrix model action. One remarkable fact is that our background, though slightly different from the plane-wave background, still permits us to get the exact action to all orders of fermionic coordinates. This fact shows that although these configurations does not keep full supersymmetry, they are easier to deal with than the ordinary curved spacetime. It would be nice to investigate these configurations and their matrix models more thoroughly. In this paper, we studied some fuzzy sphere like classical solution and found they share the same property uncovered in [8]. It would be interesting to study 1-loop [26, 27, 28], brane creation[29] issues in these backgrounds.

Our discussion focused on the configurations (1,3), which is a special class of the general supersymmetric time-dependent configurations (50,51) in the

appendix. The study of the above sections can be generalized to the general backgrounds straightforwardly. Especially, the argument of the exactness still make sense. This can be seen from the explicit form of the metric, orthogonal frame, spin connections and field strength. This means that the matrix model action in the configurations (50,51) would be exact to all order of the fermionic coordinates. It deserves more study.

Our construction shed some light on the relation between membrane regularization method and usual DCLQ prescription to construct the matrix model action. It turns out that the membrane regularization method is quite effective. It should be straightforward to generalize the method to the construction of matrix string action. In [8], it has been shown the equations of motion of the membrane and the fuzzy sphere is the same. This suggests that the matrix string action there could be obtained by matrix regularization of membrane action.

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Appendix

In this appendix, we collect some relations on the configurations discussed in [9]. The general supersymmetric time-dependent backgrounds have a metric of form

$$ds^2 = 2A_0(u)dudv + B_{ij}(u)x^i x^j (du)^2 + A_i(u)(dx^i)^2 + A_{ij}(u)x^j dx^i du, \quad (50)$$

with $B_{ij}(u) = B_{ji}(u)$ and $A_{ij}(u) = -A_{ji}(u)$, and have the field strength

$$F_{u123} = f_0(u). \quad (51)$$

The metric (50) allows an orthogonal frame

$$e^+ = \sqrt{A_0(u)}du \quad (52)$$

$$e^- = \sqrt{A_0(u)}dv + \frac{B_{ij}(u)x^i x^j}{2\sqrt{A_0(u)}}du + \frac{A_{ij}(u)x^j}{2\sqrt{A_0(u)}}dx^i \quad (53)$$

$$e^I = \sqrt{A_i(u)}dx^i \delta_i^I. \quad (54)$$

The corresponding spin connections are

$$\begin{aligned}
\phi^{-+} &= -\frac{\partial_u \sqrt{A_0}}{\sqrt{A_0}} du \\
\phi^{+i} &= 0 \\
\phi^{ij} &= -\frac{A_{ji}}{2\sqrt{A_i A_j}} du \\
\phi^{-i} &= \frac{1}{\sqrt{A_i}} \left(\frac{B_{ij} x^j}{\sqrt{A_0}} - \frac{\partial_u A_{ij} x^j}{2\sqrt{A_0}} + \frac{\partial_u \sqrt{A_0}}{A_0} A_{ij} x^j \right) du \\
&\quad - \frac{\partial_u \sqrt{A_i}}{\sqrt{A_0}} dx^i + \sum_{j \neq i} \frac{A_{ji}}{2\sqrt{A_0 A_i}} dx^j.
\end{aligned} \tag{55}$$

With the field strength, we have

$$\mathcal{O}_v = 0 \tag{56}$$

$$\mathcal{O}_u = -\frac{1}{12}(\Gamma^{+-123} + \Gamma^{123}) \frac{f_0}{\sqrt{A_1 A_2 A_3}} \tag{57}$$

$$\mathcal{O}_i = \frac{1}{24}(3\Gamma^{123}\Gamma^i + \Gamma^i\Gamma^{123})\Gamma^+ \frac{\sqrt{A_i} f_0}{\sqrt{A_0 A_1 A_2 A_3}}. \tag{58}$$

The Ricci tensor has the only nonvanishing component

$$R_{uu} = \sum_i \frac{\sqrt{A_0}}{\sqrt{A_i}} \left(-\partial_u \left(\frac{\partial_u \sqrt{A_i}}{\sqrt{A_0}} \right) - \frac{1}{\sqrt{A_0 A_i}} B_{ii} + \frac{\partial_u \sqrt{A_0} \partial_u \sqrt{A_i}}{A_0} + \sum_{j \neq i} \frac{A_{ij}^2}{4A_j \sqrt{A_0 A_i}} \right). \tag{59}$$

The nontrivial equation of motion is

$$R_{uu} = \frac{f_0^2}{2A_1 A_2 A_3} \tag{60}$$

References

- [1] H. Liu, G. Moore and N. Seiberg, Strings in a time-dependent Orbifold, JHEP 0206 (2002) 045 [hep-th/0204168]; Strings in time-dependent Orbifolds, JHEP 0210 (2002) 031 [hep-th/0206182].
L. Cornalba and M. S. Costa, Time Dependent Orbifolds and String Cosmology, Fortsch. Phys. 52, 145 (2004) [hep-th/0310099].

- [2] G. T. Horowitz and J. Polchinski, Instability of Spacelike and Null Orbifold Singularities, Phys. Rev. **D66** (2002) 103512 [hep-th/0206228].
A. Lawrence, On the Instability of 3d Null Singularities, JHEP 0211 (2002) 019 [hep-th/0205288].
- [3] J. McGreevy and E. Silverstein, The Tachyon at the End of the Universe, JHEP **0508**, 090 (2005) [hep-th/0506130]; E. Silverstein, Dimensional Mutation and Spacelike Singularities, Phys. Rev. D **73**, 086004 (2006) [hep-th/0510044]; G. T. Horowitz and E. Silverstein, The Inside Story: Quasilocal Tachyons and Black Holes, Phys. Rev. D **73**, 064016 (2006) [hep-th/0601032].
- [4] B. Craps, S. Sethi and E. P. Verlinde, A Matrix Big Bang, JHEP 0510, (2005) 005 [hep-th/0506180].
- [5] M. Li, A Class of Cosmological Matrix Models, Phys. Lett. **B626** (2005) 202-208, [hep-th/0506260].
- [6] M. Li, W. Song, Shock Waves and Cosmological Matrix Models, JHEP 0510 (2005) 073 [arXiv:hep-th/0507185].
- [7] Y. Hikida, R. R. Nayak, K. L. Panigrahi, D-branes in a Big Bang/Big Crunch Universe: Misner Space, JHEP 0509 (2005) 023, [hep-th/0508003].
- [8] S. R. Das and J. Michelson, pp Wave Big Bangs: Matrix Strings and Shrinking Fuzzy Spheres, Phys. Rev. D **72**, (2005) 086005 [hep-th/0508068].
- [9] B. Chen, The Time-dependent Supersymmetric Configurations in M-theory and Matrix Models, Phys. Lett. **B632** (2006) 393-398, [hep-th/0508191].
- [10] J. H. She, A Matrix Model for Misner Universe and Closed String Tachyons, JHEP 0601 (2006) 002, [hep-th/0509067].
- [11] B. Chen, Y. I. He and P. Zhang, Exactly Solvable Model of Superstring in Plane-Wave Background with Linear Null Dilaton, Nucl. Phys. B **741**, (2006) 269 [hep-th/0509113].

- [12] T. Ishino, H. Kodama, N. Ohta, Time-dependent Solutions with Null Killing Spinor in M-theory and Superstrings, Phys. Lett. **B631** (2005) 68-73, [hep-th/0509173].
- [13] D. Robbins, S. Sethi, A Matrix Model for the Null-Brane, JHEP 0602 (2006) 052, [hep-th/0509204].
- [14] C. S. Chu, P. M. Ho, Time-dependent AdS/CFT Duality and Null Singularity, [hep-th/0602054].
- [15] S. R. Das, J. Michelson, K. Narayan, S. P. Trivedi, Time Dependent Cosmologies and Their Duals, [hep-th/0602107].
- [16] F. L. Lin, W. Y. Wen, Supersymmetric Null-like Holographic Cosmologies, [hep-th/0602124].
- [17] W. Taylor, M. Van Raamsdonk, Multiple D0-branes in Weakly Curved Backgrounds, Nucl. Phys. **B558** (1999) 63-95, [hep-th/9904095]; Multiple Dp-branes in Weak Background Fields, Nucl. Phys. **B573** (2000) 703-734, [hep-th/9910052].
- [18] T. Banks, W. Fischler, S.H. Shenker, L. Susskind, M Theory as a Matrix Model: a Conjecture, Phys. Rev. D55 (1997) 5112-5128, [hep-th/9610043].
- [19] B. de Wit, J. Hoppe and H. Nicolai, On the Quantum Mechanics of Supermembranes, Nucl. Phys. **B305** [FS23] (1988), 545.
- [20] D. Berenstein, J. Maldacena and H. Nastase, Strings in Flat Space and pp-waves from $N = 4$ Super-Yang-Mills, JHEP 0204 (2002) 013 [hep-th/0202021].
- [21] K. Dasgupta, M. M. Sheikh-Jabbari, M. Van Raamsdonk, Matrix Perturbation Theory for M-theory on a PP-Wave, JHEP 0205 (2002) 056, [hep-th/0205185].
- [22] B. de Wit, K. Peeters and J. Plefka, Superspace Geometry for Supermembrane Backgrounds, Nucl. Phys. B **532**, (1998) 99 [hep-th/9803209].
- [23] S. R. Das, J. Michelson, A. D. Shapere, Fuzzy Spheres in pp Wave Matrix String Theory, Phys. Rev. **D70**(2004) 026004, [hep-th/0306270].

- [24] N. Iizuka, Supergravity, Supermembrane and M(atrix) Model on pp-Waves, Phys. Rev. D **68**, (2003) 126002 [hep-th/0211138].
- [25] J.G. Russo, A.A. Tseytlin, A Class of Exact pp-Wave String Models with Interacting Light-Cone Gauge Actions, JHEP 0209 (2002) 035, [hep-th/0208114].
- [26] M. Li, W. Song, A One Loop Problem of the Matrix Big Bang Model, [hep-th/0512335].
- [27] B. Craps, A. Rajaraman, S. Sethi, Effective Dynamics of the Matrix Big Bang, [hep-th/0601062].
- [28] E. J. Martinec, D. Robbins, S. Sethi, Toward the End of Time, [hep-th/0603104].
- [29] S. R. Das, J. Michelson, Matrix Membrane Big Bangs and D-brane Production, [hep-th/0602099].